An Interval-based Framework for Clock Rate Synchronization

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Problem Formulation

Clock State

$C_p : t \to T_p$

ideal: $C_p(t) = t$

Clock Rate

$v_p(t) = \frac{dC_p(t)}{dt}$

ideal: $v_p(t) = 1$

Clock Rate Synchronization:

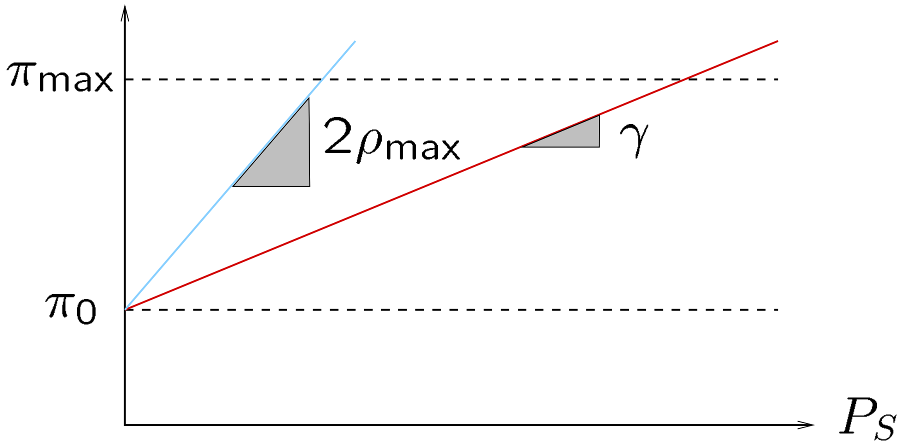
Consonance Requirement: $|v_p(t) - v_q(t)| \leq \gamma$

Drift Requirement: $|v_p(t) - 1| \leq \delta$

nodes p, q are non-faulty up to real time t

Significance of Rate Synchr.

- Duration Measurement
- Clock State Synchronization



- Fault Detection
- Initial Clock Synchronization

Relation to other Work

- Marzullo '84

  $\rightarrow$ Recognition of Clock Rate Synchr.

  $\rightarrow$ Interval-based Algorithm

- Srikanth/Toueg '87

  $\rightarrow$ cannot be better than Physical Clocks

- Schmid/Schossmaier '97

  $\rightarrow$ Framework for Clock State Synchr.

  $\rightarrow$ Interval-based Analysis Techniques

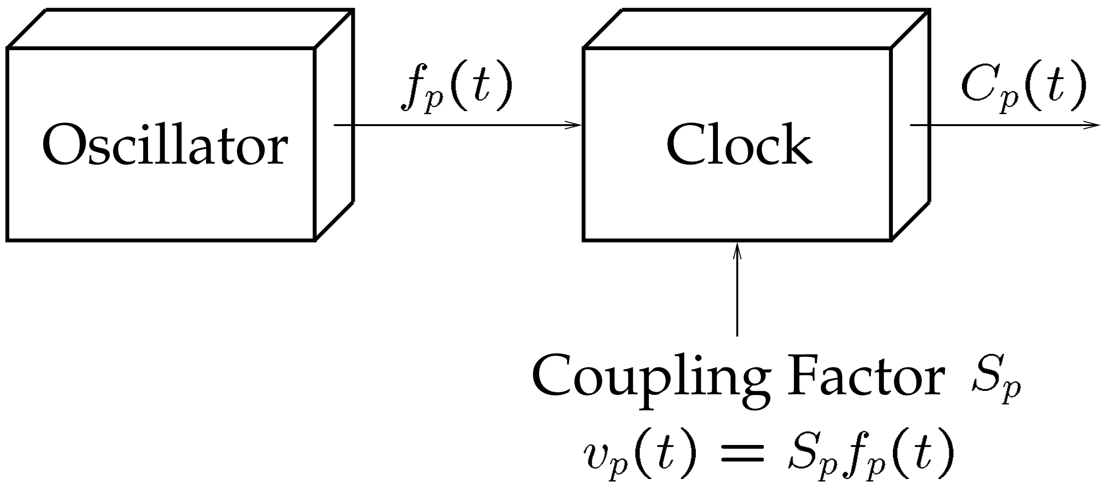
Our Approach

- Realistic System Model
- Interval Paradigm
- Building Blocks for Rate Synchr.
- Generic Algorithm
- Convergence/Validation Function
- Rigorous Analysis
- Future Research Issues

System Modeling

- Local Clock
  - rate/state adjustable
  - oscillator driven
  - precision π_{max}
- Communication Subsystem
  - packet-based
  - (un)reliable broadcast
  - delivery uncertainty ϵ_{max}
- Processor
  - execution time η_{max}

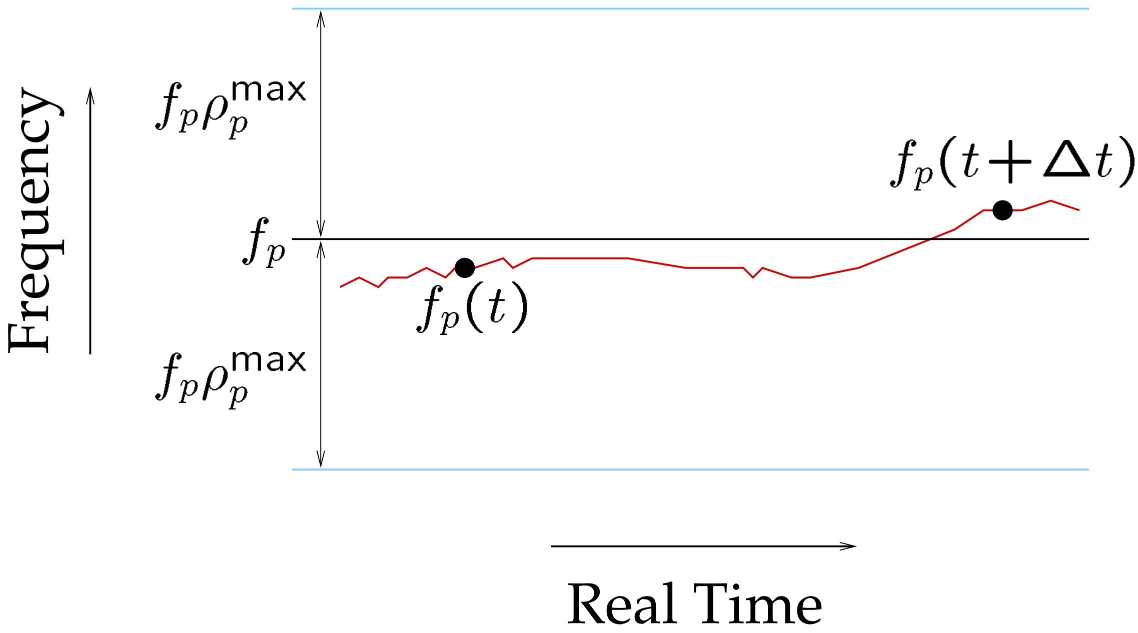
Oscillator-Clock Coupling



without Rate Synchronization:
S_p is fixed to $1/f_p$

with Rate Sycnhronization:
S_p is adjustable in the range $1/f_p(1 \pm \rho_p^{\mathsf{max}})$
e.g., UTCSU-ASIC

Oscillator Frequency



Deterministic Influences:
temperature, aging, pressure, shock, ...

Stochastic Influences:
white noise, flicker noise, random noise, ...

Oscillator Assumptions

- Drift Assumption:

$$\left|\frac{f_p(t)}{f_p} - 1\right| \leq \rho_p^{\mathsf{max}} \qquad \forall t \geq t_0$$

$\rho_p^{\mathsf{max}} \ldots$ max. oscillator drift [ppm]

- Stability Assumption:

$$\left|\frac{f_p(t+\Delta t)}{f_p(t)} - 1\right| \leq \sigma_p \Delta t \qquad \forall t \geq t_0$$

$\sigma_p \ldots$ oscillator stability [ppm/s]

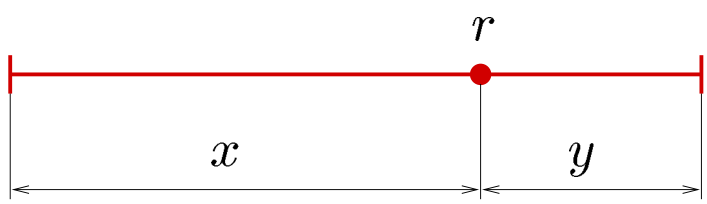
for useful $0 \leq \Delta t \leq \infty$

Oscillator Examples

Oscillator	ρ_p^{max} [1]	σ_p [1/s]
Uncomp. Quartz (XO)	10^{-5}	10^{-7}
TCXO	10^{-6}	10^{-9}
OCXO	10^{-7}	10^{-10}
Rubidium Gas Cell	10^{-10}	10^{-11}
Cesium Beam	10^{-11}	10^{-11}

Building Blocks

Intervals $\boldsymbol{I} = [x, r, y]$ or $\boldsymbol{I} = [r \pm x]$



- Rate Intervals $\mathbf{R}_p$ and $\mathbf{R}_{p,q}$
  $\rightarrow$ information about v_p

- Quotient Rate Intervals $\mathbf{Q}_{p,q}$
  $\rightarrow$ information about $\frac{v_q}{v_p}$

- Consonance Intervals γ
  $\rightarrow$ information about $|v_p - v_q|$

Local Rate Interval

$\mathbf{R}_p = [\theta_p^-, 1, \theta_p^+]$ is correct during $[t_1, t_2]$ iff

$$1 \in v_p(t)\mathbf{R}_p \qquad \forall t_1 \leq t \leq t_2$$

clock drift δ_p $\Rightarrow$ $\mathbf{R}_p = \left[\frac{\delta_p}{1+\delta_p}, 1, \frac{\delta_p}{1-\delta_p}\right]$ is correct

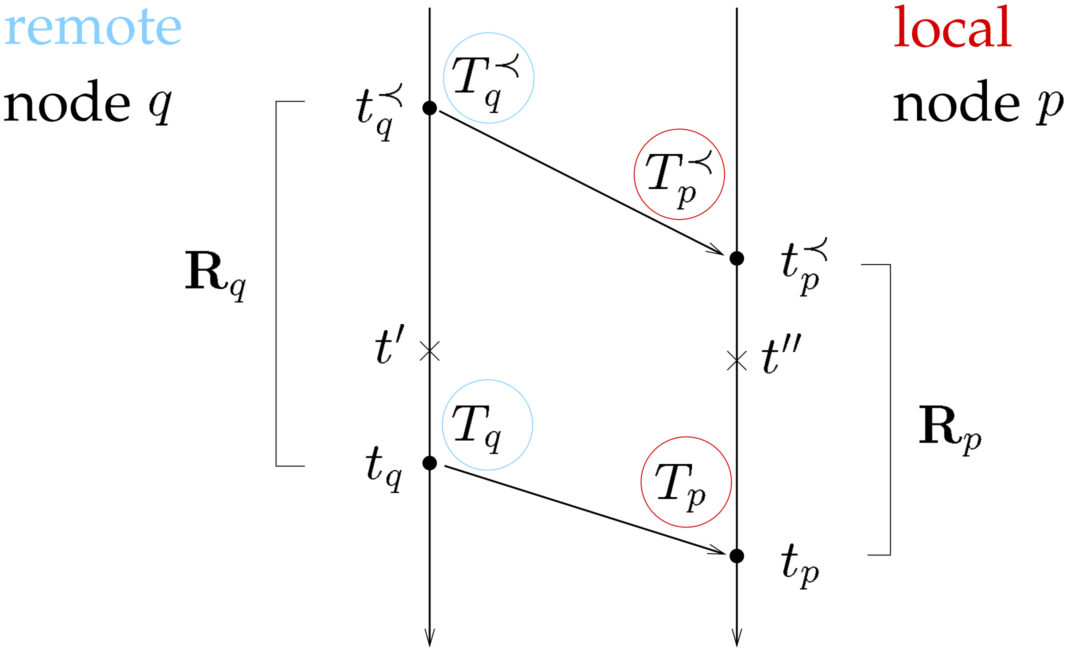
$\mathbf{R}_p$ is correct $\Rightarrow$ $\delta_p = \max\left\{\frac{\theta_p^+}{1+\theta_p^+}, \frac{\theta_p^-}{1-\theta_p^-}\right\}$

If $\mathbf{R}_p$ is correct at T_1, then

$$\mathbf{R}_p + \left[0 \pm \frac{\sigma_p(T_2-T_1)}{1-2||\mathbf{R}_p||}\right]$$

is correct at T_2 when no Resynchr. occurred

Relative Rate Measurement



Quotient Rate Interval: $\frac{v_q(t')}{v_p(t'')} \in \mathbf{Q}_{p,q}$

$$\mathbf{Q}_{p,q} = \left[\frac{T_q - T_q^{\prec}}{T_p - T_p^{\prec}} \pm \left(\frac{(\sigma_p + \sigma_q)(T_q - T_q^{\prec})}{2} + \frac{\epsilon\mathrm{max}}{T_p - T_p^{\prec}}\right)\right]$$

Remote Rate Interval

remote
node q

local
node p

local
rate interval
$\mathbf{R}_q$
is correct at $t_q^{\prec}$

$\longrightarrow$

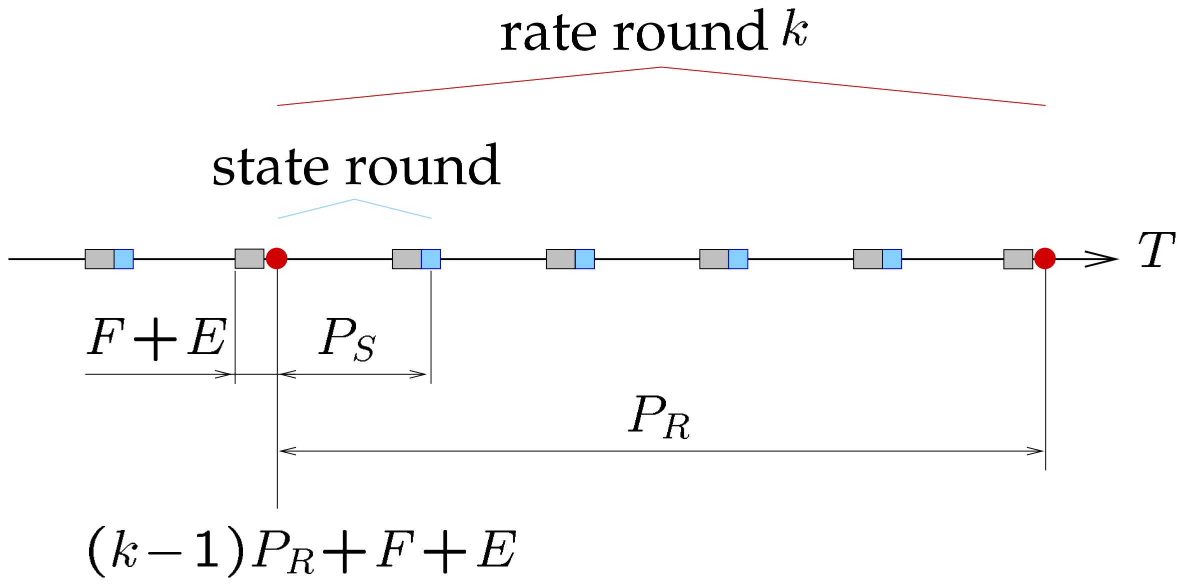
remote
rate interval
$\mathbf{R}_{p,q}$
is correct at t_p

$$\mathbf{R}_{p,q} = \mathbf{Q}_{p,q} \cdot \mathbf{R}_q$$

$$\mathsf{ref}(\mathbf{R}_{p,q}) \in \left[1 \pm \left(||\mathbf{R}_p + \mathbf{R}_q|| + \frac{\sigma_p + \sigma_q}{2} P_R + \frac{\epsilon_{\mathsf{max}}}{P_R}\right)\right]$$

$$\mathsf{align}(\mathbf{R}_{p,q}) \subseteq \mathsf{align}(\mathbf{R}_q) + \left[0 \pm \left(\frac{\sigma_p + \sigma_q}{2} P_R + \frac{\epsilon_{\mathsf{max}}}{P_R}\right)\right]$$

Round Structure



- State Resynchr.
- Rate+State Resynchr.
- Full Message Exchange and Computation

Algorithm CRA

Each node p performs:

at kP_R : broadcast

  timestamped packet(s) $\langle \mathbf{R}_p, U_p, T_p \rangle$

until $kP_R + F$: receive

  packets $\langle \mathbf{R}_q, U_q, T_q \rangle$ from nodes q

  timestamp them with $T_{p,q}$

at $kP_R + F$: compute

  quotient rate intervals $\mathbf{Q}_{p,q}$

  remote rate intervals $\mathbf{R}_{p,q}$

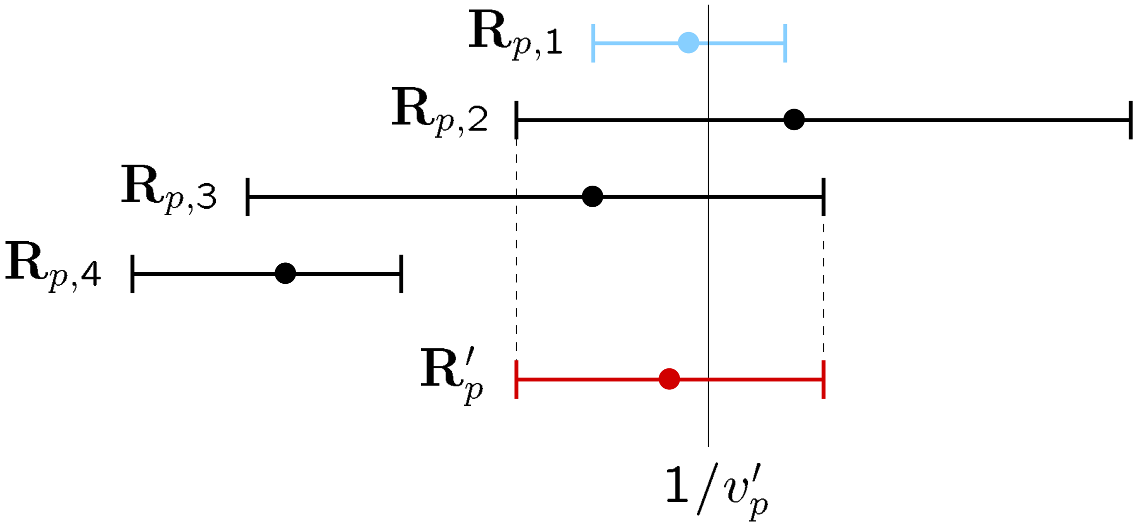
  $\mathbf{R}'_p$ via $\mathcal{CV}_{\mathcal{F}}(\cdot)$ and/or $\mathcal{VAL}_{\mathcal{F}}(\cdot)$

  new local rate interval $\mathbf{R}_p$

at $kP_R + F + E$: adjust

  clock rate by setting $S_p \leftarrow \mathrm{ref}(\mathbf{R}'_p) S_p$

Rate Adjustment



best approximation of v'_p is $1/\mathrm{ref}(\mathbf{R}'_p)$

$\Rightarrow$ new clock rate and coupling factor

$v_p = v'_p \mathrm{ref}(\mathbf{R}'_p)$ via $S_p = S'_p \mathrm{ref}(\mathbf{R}'_p)$

$\Rightarrow$ new local rate interval

$\mathbf{R}_p = \mathbf{R}'_p/\mathrm{ref}(\mathbf{R}'_p) = \mathrm{norm}(\mathbf{R}'_p)$

Convergence Function

$$\mathbf{R}'_p = \mathcal{CV}_{\mathcal{F}}\left(\mathbf{R}_{p,1}, \ldots, \mathbf{R}_{p,n}\right)$$

has to ensure

- correctness of $\mathbf{R}'_p$
- drift preservation
- consonance preservation
- consonance enhancement

in spite of faulty $\mathbf{R}_{p,q}$'s

Examples:
plain Intersection, Marzullo's function,
Orthogonal-Accuracy function (Schmid '97)

Abstract Fault Model

we ignore actual causes and focus on $\mathbf{R}_{p,q}$, thus abstract fault model $\mathcal{F}$

can be faulty in terms of their reference point, length, or missing

Classification

on a single node:

- miss
- timing
- value

on a node pair:

- crash
- omission
- restriced
- abritrary

Validation Function

Primary nodes with GPS-Receiver(s) $\longrightarrow \mathcal{X}_p$

Non-primary nodes $\longrightarrow \mathcal{R}_p$

Solutions for External Rate Synchronization:

- Clock Validation (Schmid '94)

  if $\mathbf{X}'_p = \mathcal{CV}_{\mathcal{F}}(\mathcal{X}_p)$ is consistent with $\mathbf{R}'_p = \mathcal{CV}_{\mathcal{F}}(\mathcal{R}_p)$ then $\mathbf{R}'_p = \mathbf{X}'_p$

- new Approaches

  $$\mathbf{R}'_p = \mathcal{VAL}_{\mathcal{F}}(\mathcal{R}_p; \mathcal{X}_p; \mathcal{X}_p^{(-1)}; \mathcal{X}_p^{(-h)})$$

Consonance Intervals

$\{\mathbf{R}_1, \ldots, \mathbf{R}_n\}$ is $\boldsymbol{\gamma}$–consonant during $[t_1, t_2]$ iff

$$\bigcap_{p=1}^{n} v_p(t)(\mathsf{ref}(\mathbf{R}_p) + \boldsymbol{\gamma}) \neq \emptyset \qquad \forall t_1 \leq t \leq t_2$$

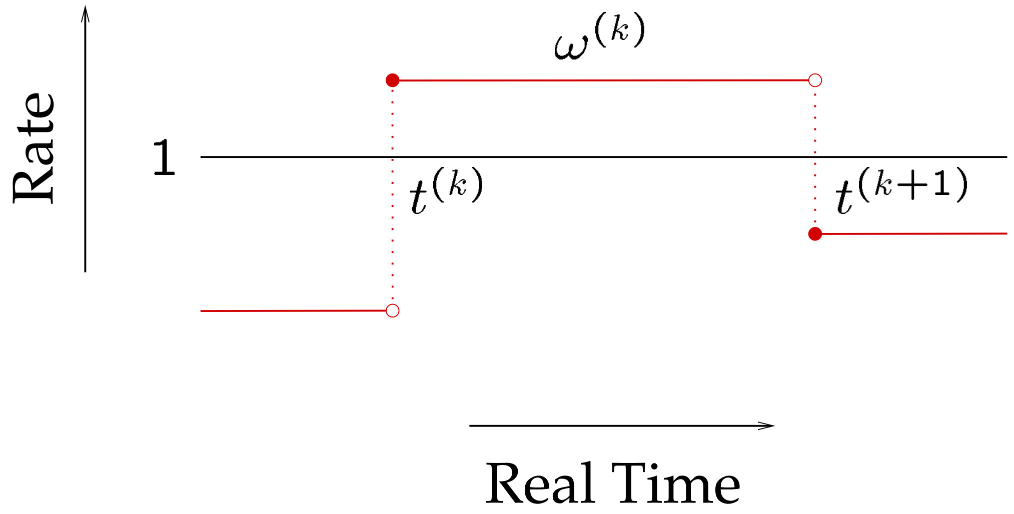
ensemble has consonance γ, clock drift δ	$\Rightarrow$	$\{\mathbf{R}_1, \ldots, \mathbf{R}_n\}$ is $\boldsymbol{\gamma}$–consonant for any $\boldsymbol{\gamma} \supseteq \left[0 \pm \frac{\gamma}{2(1-\delta)}\right]$
$\{\mathbf{R}_1, \ldots, \mathbf{R}_n\}$ is $\boldsymbol{\gamma}$–consonant, clock drift δ	$\Rightarrow$	ensemble has consonance of at most $\gamma = (1+\delta)\|\|\boldsymbol{\gamma}\|\|$

Internal Global Rate

Let $\{\mathbf{R}_1, \ldots, \mathbf{R}_n\}$ be $\gamma-$consonant at $t^{(k)}$.

Define $\omega(t) = \omega^{(k)}$ for $t^{(k)} \leq t < t^{(k+1)}$ with $\omega^{(k)}$ taken arbitrary from

$$\bigcap_{p=1}^{n} v_p(t^{(k)})(\mathsf{ref}(\mathbf{R}_p) + \gamma)$$



γ-Correctness

$\mathbf{R}_p$ is γ_p–correct w.r.t. $\omega(t)$ during $[t_1, t_2]$ iff

$$\omega(t) \in v_p(t)(\mathsf{ref}(\mathbf{R}_p) + \gamma_p) \qquad \forall t_1 \leq t \leq t_2$$

Consonance Intervals γ_p behave like Rate Intervals, hence they can be

$\rightarrow$ considered separately!

$\rightarrow$ deteriorated locally, and

$\rightarrow$ transferred between nodes
by keeping up their correctness w.r.t. $\omega(t)$

Interval Dissemination

at $t^{(k)}$

at $t^{(k+1)}$

$$\mathbf{R}_i \text{ is } \gamma^{(k)}-\text{correct} \quad \begin{array}{l} \nearrow \ \mathbf{R}_{p,i} \text{ is } \gamma_{p,i}-\text{correct} \\ \qquad\qquad \text{are } \gamma_I-\text{consonant} \\ \searrow \ \mathbf{R}_{q,i} \text{ is } \gamma_{q,i}-\text{correct} \end{array}$$

- $\gamma_{p,i}, \gamma_{q,i} \subseteq \gamma_H$, where

  $\gamma_H \subseteq \gamma^{(k)} + \left[0 \pm \left(\sigma_{\mathsf{max}} P_R + \frac{\epsilon_{\mathsf{max}}}{P_R}\right)\right]$

- $\gamma_I \subseteq \left[0 \pm \left(\sigma_{\mathsf{max}} P_R + \frac{\epsilon_{\mathsf{max}}}{P_R}\right)\right]$

Characterization of $\mathcal{CV}_{\mathcal{F}}(\cdot)$

like Schneider '87 or Schmid/Schossmaier '97

Provided that

$$\mathbf{R}'_p = \mathcal{CV}_{\mathcal{F}}\,(\mathbf{R}_{p,1}, \ldots, \mathbf{R}_{p,i}, \ldots, \mathbf{R}_{p,n})\ \text{and}$$

$$\mathbf{R}'_q = \mathcal{CV}_{\mathcal{F}}\,(\mathbf{R}_{q,1}, \ldots, \mathbf{R}_{q,i}, \ldots, \mathbf{R}_{q,n}),$$

a weakly monotonic, translation invariant Convergence Funct. $\mathcal{CV}_{\mathcal{F}}(\cdot)$ with respect to Fault Model $\mathcal{F}(\cdot)$ is characterized by its

(1) Consonance Preservation Funct. $\mathcal{CP}(\cdot)$

(2) Consonance Enhancement Funct. $\mathcal{CE}(\cdot)$

(3) Drift Preservation Funct. $\mathcal{DP}^{\pm}(\cdot)$

Consonance Preservation Funct.

$\mathcal{CP}(\cdot)$ is weakly monotonic, and specifies that

$\mathbf{R}'_p$ is $\mathcal{CP}(\gamma_{p,1},\ldots,\gamma_{p,n};\gamma_H;\gamma_I;\ldots)-$correct

with $||\mathcal{CP}(\gamma^1,\ldots,\gamma^n;\gamma_H;\gamma_I;\ldots)|| = \mathcal{O}(||\gamma_H||)$

useful to determine the maximum amount of Rate Adjustment for a non-faulty node p

$$\frac{S_p^{(k+1)}}{S_p^{(k)}} \in 1+\gamma^{(k)}+[0\pm\sigma_p P_R]+ \mathsf{swap}(\mathcal{CP}(\gamma_{p,1},\ldots,\gamma_{p,n};\gamma_H;\gamma_I;\ldots))$$

Consonance Enhancement Funct.

$\mathcal{CE}(\cdot)$ is weakly monotonic, and specifies that

$\{\mathbf{R}'_p, \mathbf{R}'_q\}$ is $\gamma'-$consonant satisfying

$$||\gamma'|| = \mathcal{CE}(\gamma^1, \ldots, \gamma^n; \gamma_H; \gamma_I; \ldots) < \mathcal{O}(||\gamma_H||)$$

useful to determine the bounding Consonance Interval γ^* by solving equation

$$||\gamma^*|| = \mathcal{CE}(\gamma^1, \ldots, \gamma^n; \gamma_H; \gamma_I; \ldots),$$

since $\mathbf{R}_p$ is $\gamma^{(k)}-$correct with $\gamma^{(k)} \subseteq \gamma^*$ at the beginning of round $k \geq 1$, when $\gamma^{(0)} \subseteq \gamma^*$.

Convergence Funct. Example

the Orthogonal-Accuracy function $\mathcal{OA}(\cdot)$ devised in Schmid '97 used for Clock Rate Synchronization is characterized by

- $\mathcal{CP}(\gamma_H) = \gamma_H$
- $\mathcal{CE}(\gamma_H, \gamma_I) = \frac{||\gamma_H + \gamma_I||}{2}$

which yields

- $||\gamma^*|| = 2\left(\sigma_{\text{max}} P_R + \frac{\epsilon_{\text{max}}}{P_R}\right)$
- $\frac{S_p^{(k+1)}}{S_p^{(k)}} \in \left[1 \pm \left(6\sigma_{\text{max}} P_R + 5\frac{\epsilon_{\text{max}}}{P_R}\right)\right]$

Notable: $P_R^{\text{opt}} \approx \sqrt{\frac{\epsilon_{\text{max}}}{\sigma_{\text{max}}}}$

Drift Preservation Funct.

$\mathcal{DP}^{\pm}(\cdot)$ are weakly monotonic, and specify that

$$\mathsf{align}(\mathbf{R}'_p) \subseteq [\mathcal{DP}^{-}(\mathbf{V}_{p,1}, \ldots, \mathbf{V}_{p.n}; \gamma_H; \gamma_I; \ldots), 0 \\ \mathcal{DP}^{+}(\mathbf{V}_{p,1}, \ldots, \mathbf{V}_{p.n}; \gamma_H; \gamma_I; \ldots)]$$

when $\mathsf{align}(\mathbf{R}_{p,i}) \subseteq \mathbf{V}_{p,i}$

useful to determine bounding Rate Intervals $\mathbf{V}_p^{(k)}$ such that

$$\mathsf{align}(\mathbf{R}_p) \subseteq \mathbf{V}_p^{(k)}$$

at the beginning of round $k \geq 1$, when initially $\mathsf{align}(\mathbf{R}_p) \subseteq \mathbf{V}_p^{(0)}$.

Summary

- Consonance/Drift Requirement
- Oscillator-Clock Model
- Stability Assumption
- Rate Intervals
- Clock Rate Synchr. Algorithm
- Similarity to Clock State Synchr.
- Interval-based Analysis

Future Work

- Integration of Clock State and Rate Synchronization
- Initialization/Joining Problem
- Convergence/Validation Functions along with Fault Models
- Experimental Evaluations
- Optimality Results for Clock Rate Synchronization